New Zealand Mathematical Olympiad Committee

March Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web in about two months time, or can be obtained from me by email earlier if you provide evidence that you've tried the problems seriously.

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1. Recall, that for a positive integer c, $c!$ denotes the product of the positive integers from 1 to c inclusive (so, for example, $4! = 1 \times 2 \times 3 \times 4$.) Determine all pairs of positive integers (m, n) where $m < n$ and $n!/m!$ is a power of 2.

2. Five cars leave Christchurch headed south on State Highway 1 to Dunedin, separated by various intervals, and traveling at various speeds. Before any of them reach Dunedin, five other cars leave Dunedin headed north on State Highway 1 to Christchurch. Whenever any two of these ten cars, traveling in opposite directions meet, they both stop and turn around. Whenever any of the cars reaches either Christchurch or Dunedin (regardless of where they started), they remain there. Assuming that only two cars ever meet simultaneously, what are the minimum and maximum possible numbers of meetings of cars before they all stop?

3. The quadrilateral $ABCD$ is a trapezoid, with $AD \parallel BC$ (and $AC \not\parallel BD$.) The point K is on AB. Prove that the line through A parallel to KC and the line through B parallel to KD intersect at a point on CD.

4. There is a well known construction that produces a sequence of 1000 consecutive integers, none of which is prime:
$$1001! + 2, 1001! + 3, \cdots 1001! + 1001$$
as the first has 2 as a factor, the second 3, the third 4, ..., and the last 1001. Is there a sequence of 1000 consecutive integers exactly seven of which are prime?

March 3, 2009

http://www.mathsolympiad.org.nz