New Zealand Mathematical Olympiad Committee

2010 April Problems

These problems are intended to help students prepare for the 2010 camp selection problems, used to choose students to attend our week-long residential training camp in Christchurch in January.

In recent years the camp selection problems have been known as the "September Problems", as they were made available in September. This year we're going to trial moving the selection problems earlier in the year, releasing them in July and moving the due date to August. This will allow more time for pre-camp training, building up to Round One of the British Mathematical Olympiad in December.

The solutions will be posted in about two month's time, but can be obtained before then by email if you write to me with evidence that you've tried the problems seriously.

Good luck!

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1. A coin has been placed at each vertex of a regular 2008-gon. These coins may be rearranged using the following move: two coins may be chosen, and each moved to an adjacent vertex, subject to the requirement that one must be moved clockwise and the other anti-clockwise. Decide whether, using this move, it is possible to rearrange the coins into
   (a) 8 heaps of 251 coins each;
   (b) 251 heaps of 8 coins each.
2. Let $ABCD$ be a convex quadrilateral that is not a parallelogram. The straight line passing through the midpoints of the diagonals of $ABCD$ intersects the sides AB and CD in the points M and N respectively. Prove that the triangles ABN and CDM have the same area.
3. Find all positive integers m and n such that $6^m + 2^n + 2$ is the square of an integer.
4. Determine all functions $f : \mathbb{N} \to \mathbb{N}$ satisfying
$$\frac{f(x+y)+f(x)}{2x+f(y)} = \frac{2y+f(x)}{f(x+y)+f(y)},$$
for all $x, y \in \mathbb{N}$.

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